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HEAT TRANSFER MODEL FOR COMPOSITE FIRST WALL MATERIALS IN A PULSED HIGH-BETA CONTROLLED THERMONUCLEAR REACTOR (CTR)

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ABSTRACT

A computer model has been constructed to predict temperature and time excursions for radial composite walls currently under consideration for pulsed high-beta Z-pinch machines. The effects of incident flux, internal heat distribution functions, thermal properties, and material dimensions have been examined for a Nb/Al_2O_3 composite to establish the feasibility of the model.

I. INTRODUCTION AND SCOPE

In a previous report,[1] a preliminary treatment of first wall heat transfer and chemical stability effects was presented. For homogeneous materials such as Nb, Al_2O_3, BeO, or BN temperature excursions and/or chemical reactivity with molecular or atomic hydrogen became prohibitive, indicating that a composite first wall might present a feasible alternative. Prediction of thermodynamic equilibrium, kinetic, thermal stressing, and radiation damage effects require first-hand knowledge of anticipated temperature-time profiles for composite wall materials intended for use in pulsed, high-beta, controlled thermonuclear reactors (CTR's) where heat fluxes on the order of 1 kW/cm^2 or more are possible. Furthermore, estimates of maximum operating temperatures for the molten lithium blanket are useful in establishing the effectiveness of proposed CTR's in producing high temperature heat sources for direct or indirect energy production.

II. DESCRIPTION OF THE MODEL

A. Basic Geometry

Due to the large radius of curvature (30 m) and torus diameter (~ 1 m) a rectangular coordinate system was used for the model. Figure 1 illustrates schematically how a Z-pinch prototype might be

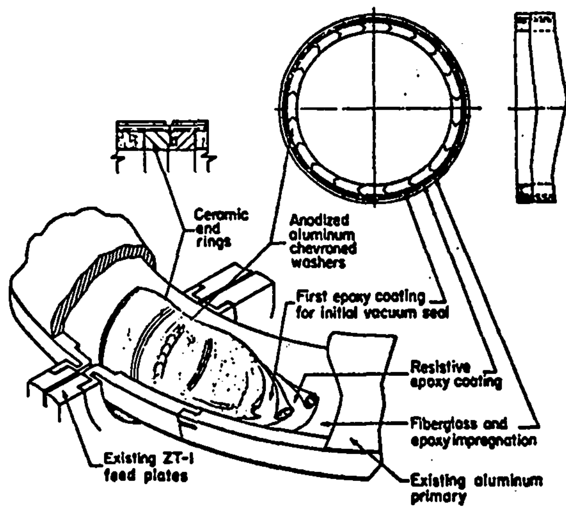

Fig. 1. Schematic of prototype Z-pinch design.[2]

designed.[2] The major feature of interest is the radial arrangement of the composite first wall. In the prototype design the conductor (material 1) is an aluminum washer separated by thin layers of anodized aluminum which can be conceptually thought of as the insulator (material 2). Figures 2A and 2B schematically represent the geometry used in the model. The grid has I2 points in the x-direction and J

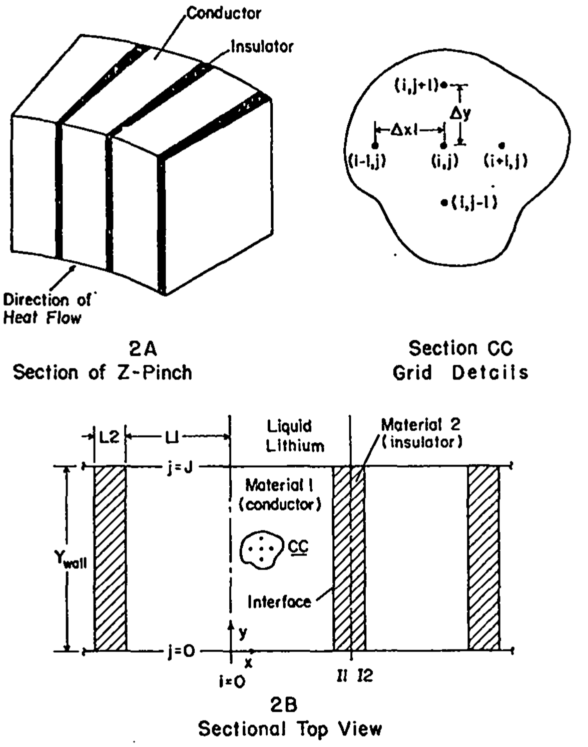

Fig. 2. Geometry employed for finite difference grid. I2 x J points having Δy spacing in the y-direction and Δx1(Δx2)spacing in the x-direction for materials 1(2).

points in the y-direction with the point at I1 on the interface between materials 1 and 2.

A time-dependent heat flux impinges on the inner surface of the composite [i=0, ..., I1, ... I2; j=0], and a liquid metal (lithium)/metal conduction temperature dependent heat transfer resistance exists on the outer surface [i=0, ..., I1, ..., I2; j=J]. The two center lines (-·-) define mirror symmetry planes in each material and can be represented by a zero flux [$- k\frac{\partial T}{\partial x} = 0$] condition.

B. Design Criteria

Heat enters the first wall via several sources, including:

1. Bremsstrahlung radiation,
2. n-γ reactions within the wall, and
3. direct neutron deposition energy.

In a preliminary report, Burnett, Ellis, Oliphant, and Ribe[3] demonstrated that most of the energy deposited (> 85%) was Bremsstrahlung energy. In our model, the total heat absorbed is divided

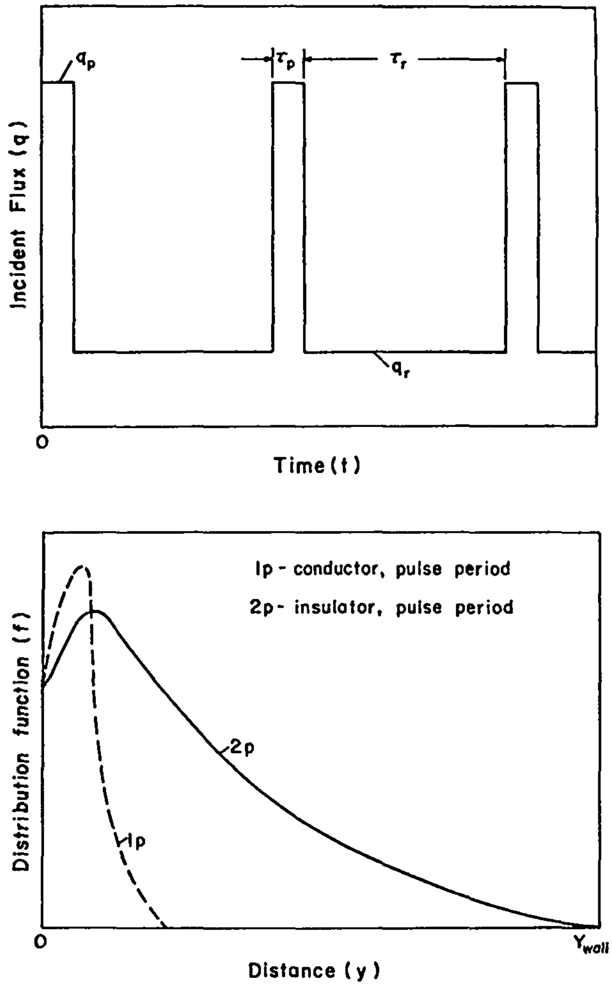

Fig. 3. Incident heat flux q and heat distribution functions $f = H(y)/q_p \, \Delta y$ expressed as a fraction of the pulse heat flux q_p (arbitrary scales).

into two quantities:

1. An incident flux which is deposited at the surface y = 0.
2. A distributed heat source function H = f(y) representing the energy absorbed as a function of distance into the wall from the point y = 0 to the extent of the wall y = Ywall.

Consequently, for a two-component composite, there would be four H functions corresponding to each material in the pulse and rest mode. In Fig. 3, we present idealizations of these heat distribution and incident flux functions used in the current approach.

Only distribution (H(y)) curves for the pulse period are shown in Fig. 3, since negligible values for the rest period are anticipated when heat transfer to the wall will be primarily by radiation and convection from the expanding plasma. As a first approximation, one might assume that $H(y)/q = 0$ during the rest period for both materials, indicating that all of the heat is deposited on the inside surface of the wall. Nevertheless, in implementing the model, the user is free to select any heat distribution function that is appropriate. For example, for our Nb/Al_2O_3 composite both rest and pulse H functions are set to zero for Nb, and a finite H used only for the pulse mode in Al_2O_3 (see Ref. 3). In general, the insulator (ceramic) would be expected to have a much wider distribution function than the conductor (metal) as is illustrated in Fig. 3.

The square wave function idealization for q is somewhat of an over-simplification of the actual case which might show an exponential increase and decrease of heat flux during the cycle.[4] However, at this stage, a square wave functionality should be adequate. Actual values for the incident heat flux q may be determined by design limitations of the materials used in the first wall. For example, the magnitude of q can be partially controlled by changing the amount of first wall surface area for a given amount of heat produced during the cycle.

C. Governing Equations and Boundary Conditions

The following partial differential equation (PDE) applicable to unsteady state, two-dimensional heat conduction was used for both materials.

$$\alpha_i \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \frac{H_i\ (y)}{\rho_i\ C_{p_i}} = \frac{\partial T}{\partial t} \ . \qquad (1)$$

$$i = 1,2 \text{ (for both materials)} \ .$$

An ambient temperature (T_B) equal to the bulk lithium temperature is assumed for the initial condition at $t = 0$. Four boundary conditions are applied to positions specified on Fig. 2B:

1. Incident heat flux at the inside surface (see Fig. 3)

   at $y = 0$ ($j = 0$), all x

   $$-ki\left(\frac{\partial T}{\partial y}\right) = q_i(t)^{\dagger} \ . \qquad (2)$$

2. Temperature dependent flux with contact resistance at the outside surface

   at $y = 0$ ($j = 0$), all x

   $$-ki\left(\frac{\partial T}{\partial y}\right) = h\ (T - T_B)^{\dagger} \qquad (3)$$

   where h is an effective heat transfer coefficient applying to the molten lithium blanket and any solid liners that might be used.

3. Continuous flux and temperature at the interface

   at $x = L1/2$ ($i = I1$), all y

   $$k1\left(\frac{\partial T}{\partial x}\right) = k2\left(\frac{\partial T}{\partial x}\right) \ . \qquad (4)$$

4. Zero flux condition at centerlines of materials 1 and 2 via symmetry

   $$\text{at } x = 0\text{: } (i = 0), \left(\frac{\partial T}{\partial x}\right) = 0 \qquad (5)$$

   $$\text{at } x = \frac{(L1 + L2)}{2} \quad (i = I2) \quad \left(\frac{\partial T}{\partial x}\right) = 0. \qquad (6)$$

In solving Eq. (1) to generate temperature profiles as functions of time, a dimensionless temperature u was defined as

$$u \equiv \frac{T - T_B}{T_B} \ , \qquad (7)$$

and finite difference equations were written to approximate the PDE. Appendix A contains a tabular presentation of these equations. A detailed description of the finite difference formulation of the boundary conditions is presented in Appendix B. An Alternating Direction Implicit (ADI) scheme was used to solve the system of equations (see Appendix C). The advantages of an implicit rather than explicit scheme should be useful in conserving machine time and in adding to the flexibility of the code.

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† In the expression ki or q_i the i = 1 or 2 depending on what material it is.

The tridiagonal algorithm and a complete listing of the Madcap V code are presented in Appendixes D and E.

III. LIMITATIONS AND APPLICATIONS OF THE MODEL

Several features of the model have been kept general; for example, various wall sizes can be used with any two materials. If the repeating thicknesses in the x-direction, L1 and L2, become much smaller than the thickness of the wall in the y-direction Y_w, the code reverts to a unidirectional (y only) calculation of temperature profiles with area average physical properties used. Any combination of incident heat flux and internal heat generation terms can be used. The outside boundary condition (all x, y= Ywall at j=J) is temperature dependent in order that an effective heat transfer coefficient can be used which combines the resistances of a liquid lithium boundary layer and any metallic and/or ceramic backing material that might be present.

The interface condition (at i=I1) can be specified by either of two procedures (see Appendix B):

1. Criteria of continuous flux at the boundary

$$-k1 \left(\frac{\partial T}{\partial x}\right) = -k2 \left(\frac{\partial T}{\partial x}\right) . \tag{4}$$

2. Criteria of continuous flux and an operable PDE at the boundary.

In using the code, large time steps should be avoided since they can cause inaccuracies as well as instabilities because of the pulsed boundary condition and the interface between materials 1 and 2. At least 10 time steps for each pulse comprise the upper limit, i.e., for a 10 ms (10^{-3} s) pulse Δt would be 1 ms. Since the rest period is usually much longer than the pulse period, e.g., 90 ms compared to 10 ms, a larger Δt could be used during this period if conserving computation time became important.

IV. PRELIMINARY RESULTS AND DISCUSSION

The main purpose of this section is to discuss preliminary results which demonstrate the feasibility of applying our heat transfer model to CTR applications.

A. Choice of a test system

A niobium (Nb) - alumina (Al_2O_3) radial composite was selected since it is currently under consideration as a first wall composite material,[3] and because its thermal properties are representative of typical metallic conductors and ceramic insulators that might be considered at a later time. Present Z-pinch design estimates will require an insulating capacity between 1 to 3 kV/cm which will control the relative dimensions of insulator (2) to conductor (1).[2] Although actual sizes have not been specified for a real operating system, a prototype experimental design utilizing anodized aluminium washers (0.0254 cm thick Al with approximately 0.0005 cm of anodized coating) is currently under construction by Phillips and associates.[2] A large scale-up from these dimensions is anticipated for future experiments and consequently a test geometry with about 1 cm width of conductor to 0.1 cm of insulator with an overall wall thickness of 1 cm was selected. Total heat flux loads on the first wall during the pulse period are expected to be the range of 0.1 to 10 kW/cm^2 consisting mainly of Bremsstrahlung and n-γ energy. Niobium, due to its high mass number, will absorb most of the plasma energy within a very thin layer (~0.01mm).[3] Alumina, on the other hand, will absorb the energy continuously with a distribution function given in Fig. 4. As suggested by Burnett et al.[3] an average electron temperature of 25 keV was selected to define the heat generation function. During the rest period, approximately 10% of the instantaneous pulse heat flux will impinge on the inside surface of the wall with no distribution within the wall ($H(y) = 0$). As a first approximation a constant value was used during the entire rest period (see Fig. 3). In order to meet the Lawson criterion a 10% duty cycle corresponding to a 0.01 s pulse and a 0.09 s rest period has been employed for the test case. A range of outside surface (y = Ywall, Fig. 2) heat transfer coefficients from h = 0.14 to 14 cal/cm^2 s K were utilized to approximate the thermal resistance anticipated from a niobium (Nb)/ boron nitride (BN) protective liner and a molten lithium boundary layer. Average values for material properties were selected at approximately 800°C, and these are tabulated in Table I for several first wall material possibilities.

A summary of the system parameters investigated is presented in Table II. Again, we would like to emphasize that our purpose at this stage was to

TABLE I

MATERIAL PROPERTIES(*)

	k	ρ	C_p	$\alpha = k/\rho C_p$
Conductors (1)	cal/(cm^2 s K/cm)	g/cm^3	cal/gK	cm^2/s
Niobium, Nb	0.158	8.57	0.0736	0.250
Molybdenum, Mo	0.350	10.20	0.0630	0.545
Insulators (2)				
Alumina, α-Al_2O_3	0.034	3.96	0.198	0.0434
Beryllia, BeO	0.835	3.00	0.50	0.0557

k-thermal conductivity ρ-density C_p-heat capacity α-thermal diffusivity

(*) Data based on information taken at ~800°C from

1. "Perry's Handbook for Chemical Engineers," 4th Ed., McGraw-Hill N.Y., (1965).
2. "Handbook of Chemistry and Physics," Chemical Rubber Publ., N.Y., 41st Ed. (1962).
3. "Thermal Physical Properties of Matter," Vols. 1-2 Eds. Touloukian, Powell, Ho, and Klemens, Plenum Publ. Corp., N.Y. (1970).

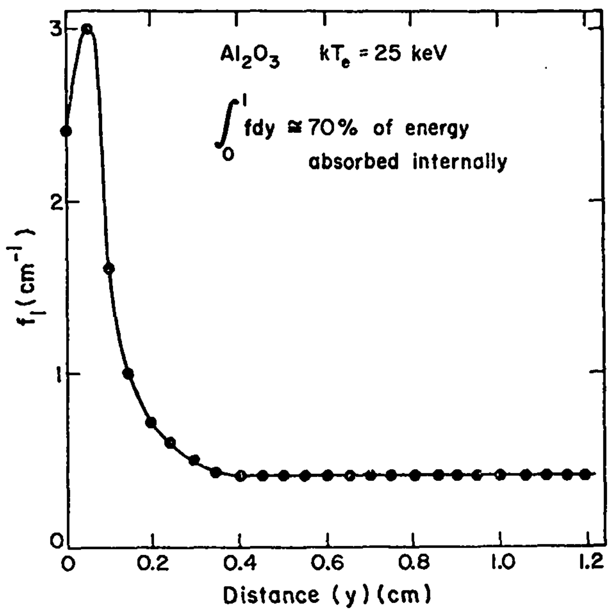

Fig. 4. Heat distribution function for Al_2O_3 for pulse period (original data Ref 3 kT_e-electron temperature of the plasma).

demonstrate calculational feasibility rather than propose a definitive design.

B. <u>Temperature-Time Excursions for a Nb/Al_2O_3 Composite</u>

Table III (A and B) provides a complete summary of the test runs made. The effects of heat flux, heat transfer coefficient, time step, and grid size parameters were all examined.

A typical temperature-time excursion for seven consecutive pulses (for complete parameter specification see Table III, Run 1) is presented in Fig. 5. Several features of the graph are apparent.

1. There are no inherent instabilities in the ADI solution.
2. The outside surface temperatures, $\Delta T(0,J)$, $\Delta T(I1,J)$, and $\Delta T(I2,J)$, do not increase due to the large value of h = 14 cal/cm^2 s K used.
3. The interface $\Delta T(I1,0)$ is between the maximum excursion in the Al_2O_3 layer ($\Delta T(0,0)$) and the minimum in Nb layer ($\Delta T(0,0)$).
4. The inside surface temperature for either material Nb or Al_2O_3 does not relax to what its initial level was before the pulse, hence there is a continuous increase in ΔT which should approach steady-state conditions after a temperature profile of sufficient magnitude has been established

TABLE II

SYSTEM PARAMETERS INVESTIGATED

1. Duty cycle τ_p = .01 s τ_r = .09 s

2. Incident heat flux

   q_i (pulse period) 0.1-1.0 kw/cm^2 (~23.82 - 238.2 cal/cm^2 s)

   q_i (rest period) .01-.1 kw/cm^2 (~2.382 - 23.82 cal/cm^2 s)

3. Heat distribution/generation function H(y)

   separate functions for insulator (2) and conductor (1) during pulse and rest mode utilized

4. Heat transfer coefficient h = .14-14 cal/cm^2 s K

   outside surface-combined resistance of backing material and liquid lithium

5. Bulk temperature T_B = 600°C[a]

6. geometrical parameters

   wall thickness Ywall = 1 cm

   Composite: conductor thickness L1 = .01-1 cm

   insulator thickness L2 = .0005 - .1 cm

7. Equation solution parameters

grid sizes	$\Delta x1$ = .0005 - .05 cm
	$\Delta x2$ = .0005 - .005 cm
	Δy = .01 - .02 cm
time steps	Δt = 10 - 2000 μs (10^{-6} s)

[a]Really arbitrary, material limitations will set the upper bound.

to conduct away the total energy deposited during the pulse and rest periods.

A series of temperature profiles are presented in Fig. 6 for the conditions of Run 5 (Table III). In this case, heat was deposited on the inside surface of the Nb layer during both pulse and rest periods and on the inside surface of the Al_2O_3 layer during the rest period. The heat distribution function given in Fig. 4 was used for Al_2O_3 during the pulse period. One can see a marked reduction in the temperature excursion of the Al_2O_3 layer caused by distributing the heat. All three profiles, at the center lines of materials 1 and 2 and the interface, are uniform in shape and magnitude for the three times given. This effect is also illustrated by comparing Fig. 7b with Fig. 8 which have identical conditions, except in Fig. 8 no heat distribution was used (H(y)'s = 0).

The magnitude of the outside surface effective heat transfer coefficient has a significant effect on predicted temperature-time excursions (see Figs. 7a and 7b). With h = 0.14 cal/cm^2 s K to approximate anticipated thermal resistances, the outside wall temperature has increased by > 60K over the bulk lithium value in 30 pulses. This ΔT will, of course, continue to increase until steady-state conditions are reached.

TABLE III

TABLE III (SECTION A)

SUMMARY OF RESULTS FOR COMPOSITE/PULSED CASE[a]

Run	Conductor (1)	Insulator (2)	Geometry: L1 (cm)	Geometry: L2 (cm)	Geometry: Ywall (cm)	Grid Size: $\Delta x1$ (cm)	Grid Size: $\Delta x2$ (cm)	Grid Size: Δy (cm)	Time Step Δt (μs)	Heat Transfer Coeff. Outside Surface h (cal/cm^2 s K)	Total Incident Flux: q_i Rest Period (kW/cm^2)	Total Incident Flux: q_i Pulse Period (kW/cm^2)
	Niobium	Alumina										
1	Nb	Al_2O_3	1.0	0.1	1.0	0.05	0.005	0.02	1000	14	0.01	1.0
2	Nb	Al_2O_3	0.01	0.0005	1.0	0.0005	0.00005	0.02	1000	14	0.01	1.0
3	Nb	Al_2O_3	1.0	0.1	1.0	0.05	0.005	0.02	1000	14	0.01	1.0
4	Nb	Al_2O_3	1.0	0.1	1.0	0.05	0.005	0.02	1000	14	0.1	1.0
5+9	Nb	Al_2O_3	1.0	0.1	1.0	0.05	0.005	0.02	1000	0.14	0.1	1.0
6	Nb	Al_2O_3	1.0	0.1	1.0	0.05	0.005	0.02	100	0.14	0.1	1.0
7	Nb	Al_2O_3	1.0	0.1	1.0	0.025	0.0025	0.01	200	0.14	0.1	1.0
8+10	Nb	Al_2O_3	1.0	0.1	1.0	0.05	0.005	0.02	1000	0.14	0.1	1.0
	Molybdenum	Beryllia										
	Mo	BeO	1.0	0.1	1.0	0.05	0.005	0.02	1000	0.14	0.1	1.0

[a] Conditions fixed for all runs: $\tau_p = 0.01$ s $\tau_r = 0.09$ s.

TABLE III (SECTION B)

SUMMARY OF RESULTS FOR COMPOSITE/PULSED CASE[a]

Run	Heat Distribution Functions Utilized[c]: Conductor (1) Pulse Period Hp1(y)	Conductor (1) Rest Period Hr1(y)	Insulator (2) Pulse Period Hp2(y)	Insulator (2) Rest Period Hp2(y)	Steady State Temperature Excursions $\Delta T(x,y,t=\infty)$[b]: Inside Surface (Plasma Side) Conductor $\Delta T(x=0, y=0, t=\infty)$ K	Inside Surface (Plasma Side) Interface $\Delta T(x=I1, y=0, t=\infty)$ K	Inside Surface (Plasma Side) Insulator $\Delta T(x=I2, y=0, t=\infty)$ K	Outside Surface Average $\Delta T(\langle x \rangle, y=Ywall, t=\infty)$ K	Comments
1	0	0	0	0	370	460	490	~ 0	
2	0	0	0	0	260	260	260	~ 0	unidirectional (y only)
3	0	0	0	0	250	320	380	~ 0	
4	0	0	Hp2(y)	0	360	351	348	~ 0	
5+9	0	0	Hp2(y)	0	650	640	640	300	
6	0	0	Hp2(y)	0	650	640	640	300[d]	
7	0	0	Hp2(y)	0	650	640	640	300[d]	
8+10	0	0	0	0	600	695	810	300	
	0	0	Hp2(y)	0					

[a] Refer to nomenclature section (Appendix F) and Figs. 1-2.

[b] Extrapolated to ∞ time.

[c] Refer to section IIC and Figs. 3-4.

[d] Equivalent to run 5.

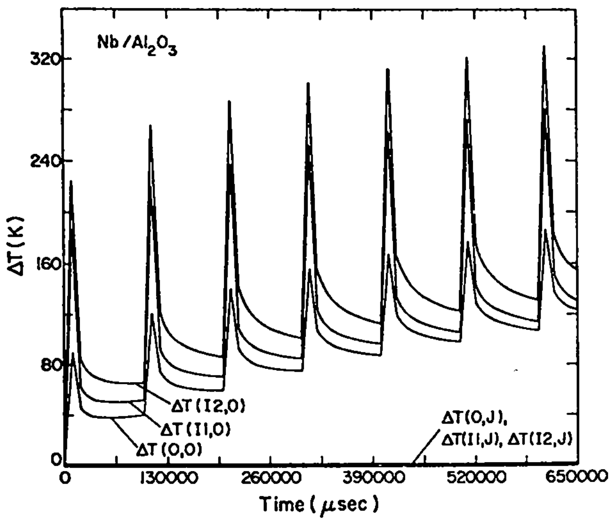

Fig. 5. Temperature-time excursions for a Nb (1cm)/Al_2O_3 (0.1 cm) composite at six locations. For parameter specifications see Table III, Run 1, and see Fig. 2 for geometrical grid locations.

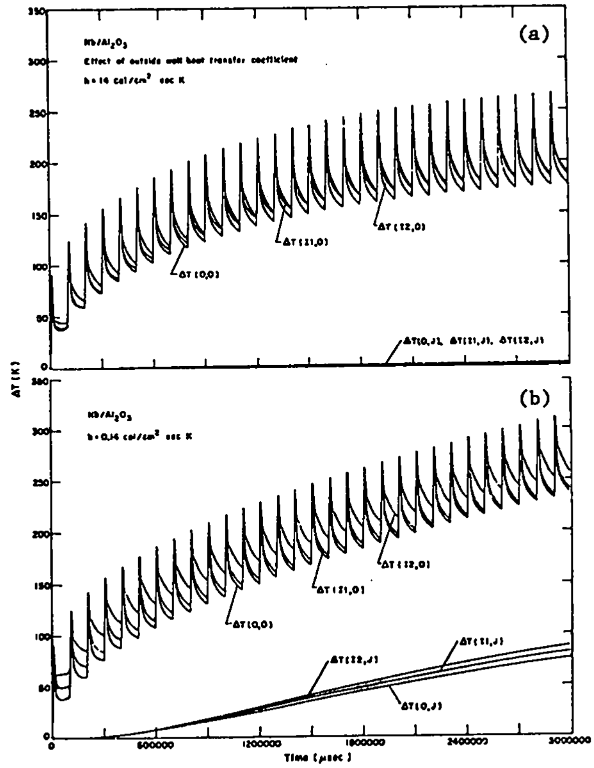

Fig. 7. Effect of outside wall heat transfer coefficient h on temperature-time excursion for a Nb/Al_2O_3 composite. For parameter specifications see Table III, Runs 4(7a), 5(7b).

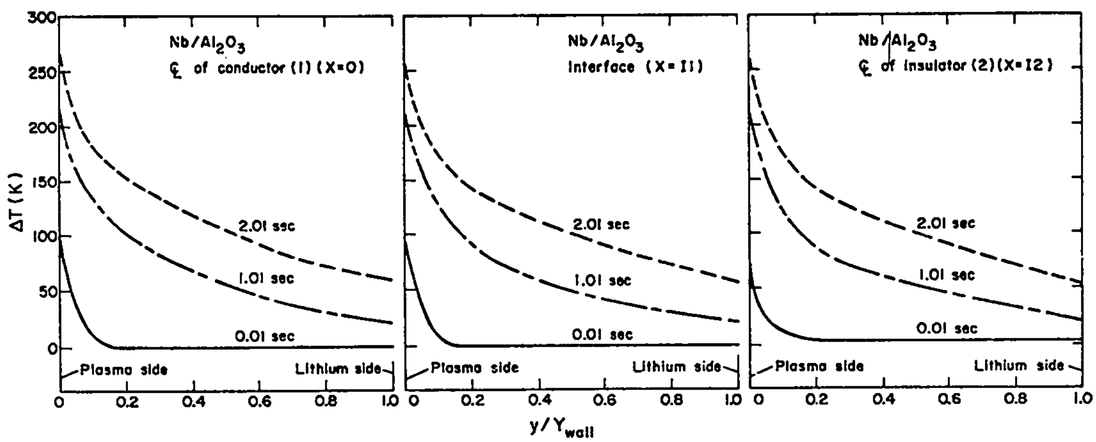

Fig. 6. Approximate temperature profiles T = f(y) at various times (2.01 s - 21 pulses, 1.01 s - 11 pulses, 0.01 s - 1 pulse). For parameter specifications see Table III, Run 5 and see Fig. 7b for temperature-time excursion.

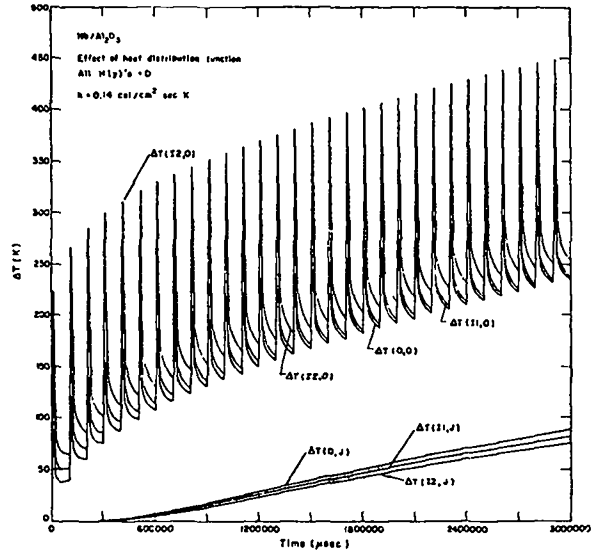

Fig. 8 Effect of heat distribution function on the temperature-time excursion of an Nb/Al_2O_3 composite. For parameter specifications see Table III, Run 8.

C. Approach to Steady State

As steady state is reached, the temperature profile at any position along the composite will stabilize except in the vicinity of the inside surface where it is continuously pulsed. This behavior was observed in a preliminary study of heat transfer effects.[1] Because the thermal time constant $\tau_w = Y_w^2/\alpha$ is large compared to a cycle time of 0.1 s, e.g., for a 1-cm wall τ_w $(Al_2O_3) \cong 23$ s and $\tau_w(Nb) \cong 6$ s and because an additional thermal resistance is imposed by the low h = .14 cal/cm^2 s K on the outside surface, successive pulsing will cause ΔT to increase at any point in the wall. A crude estimate of the maximum ΔT anticipated is given by superimposing both the ΔT_a equivalent to steady-state heat transfer through the wall and the ΔT_h caused by thermal contact resistance at the outside surface onto the ΔT_p caused by the pulse itself. For instance, at the center line of the conductor (0,0), an estimate of $\Delta T^{\infty}_{0,0}$ at steady state is given by,

$$\Delta T^{\infty}_{0,0} \cong \Delta T_a + \Delta T_h + \Delta T_p \quad ,$$

where

$$\Delta T_a = \frac{\text{(net heat transferred/time)}}{kl/Y_w}$$

$$= \frac{(q_p \tau_p + q_r \tau_r)\, Y_w}{(\tau_p + \tau_r)\, kl}$$

ΔT_p = temperature rise after the 1st pulse at (0.0)

$$\Delta T_h = \frac{\text{(net heat transferred/time)}}{h}$$

$$= \frac{(q_p \tau_p + q_r \tau_r)}{(\tau_p + \tau_r) h} \quad .$$

For the case of a 1 kW/cm^2 (238.2 cal/s cm^2) pulse and a .1 kW/cm^2 (23.82 cal/s cm^2) heat dump,

$$\Delta T_a = 287 \text{ K}$$
$$\Delta T_p \cong 90 \text{ K}$$
$$\Delta T_h = 333 \text{ K} \quad .$$

Therefore,

$$\Delta T^{\infty}_{0,0} \cong 710 \text{ K} \quad .$$

From Table III, one can see that excursions of 650 K are typical for these conditions (Runs 5,6,and 7).

D. Prototype Geometry - Effective Undirectional Heat Transport

Run 2 attempted to simulate conditions similar to those expected in the prototype Z-pinch reactor (Fig. 1). The widths of Nb and Al_2O_3 in the x-direction, .01 cm for Nb and .0005 cm for Al_2O_3, are very small compared to the thickness of the wall in the y-direction, 1 cm. Consequently, conduction in the x-direction is fast and can be neglected relative to that in the y-direction and the code performs a unidirectional ADI solution to the PDE using area average properties. In Fig. 9, temperature-time excursions are presented for the case with h = 14 cal/cm^2 s K.

E. Convergence and Stability of the Method - Effect of Grid Size and Time Step

Convergence of the ADI technique was checked with Runs 6 and 7 by reducing the grid sizes, $\Delta x1$ from .05 to .025 cm and $\Delta x2$ from .005 to .0025 cm and Δy from .02 to .01 cm, and time step Δt from

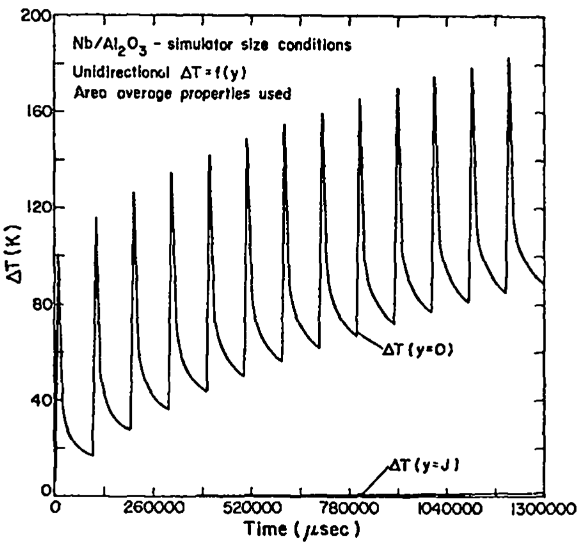

Fig. 9 Temperature-time excursion for a Nb/Al_2O_3 composite having similar dimensions to the prototype Z-pinch (Fig. 1). For parameter specifications see Table III, Run 2.

1000 to 200 μ s. Temperature profiles varied by no more than 5% at equivalent grid locations. Furthermore, when the composite was reduced to a single component, e.g., Nb, and a two-dimensional ADI solution was run, x-direction variation of ΔT was less than 0.1% and the temperature-time excursions were consistent with previous data accumulated for unidirectional heat flow using an explicit method.[1]

Although the ADI technique, as applied to rectangular two-dimensional problems, should be unconditionally stable regardless of the choices of Δt, Δx, and Δy,[9] our specific application of the ADI technique did result in instabilities as mentioned in Sec. III. The pulsed heat flux and interface condition were probably responsible for this since when they were removed from the problem by using a single component and continuous flux boundary, Δt could be selected independently of Δx and Δy. Certain improvements to the stability of the ADI procedure are obtained if the grid system is converted to a half-interval system with the interface containing $\Delta x1/2$ and $\Delta x2/2$ parts of materials 1 and 2.

F. Concluding Remarks

The computer model for heat flow in radial composite CTR first wall materials should provide a useful tool for establishing temperature excursions and profiles which are necessary in evaluating the mechanical and chemical behavior of any proposed materials.

V. RECOMMENDATIONS

1. Additional materials should be examined, including, ZrO_2, BeO, and other insulating oxides as well as Ta, Zr, Mo, and other conducting metals.
2. Having established anticipated temperature-time excursions, other properties such as chemical stability, radiation damage including void and helium bubble growth, thermal stressing, and other aspects of materials compatability should be considered.[1,5,6]
3. By selecting a range of thermal properties, dimensions, incident fluxes, and heat distribution functions, generalized thermal history charts applicable to pulsed-high-beta machines could easily be generated for use in preliminary design work.

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APPENDIX A

FINITE DIFFERENCE EQUATION FORMALISM

Tables A-1 and A-2 list the difference equations utilized by the code. Both sequences of sweeping x first and then y, and vice versa, are presented. In addition, two different equations applying at the interface between materials 1 and 2 are included. A complete description of the nomenclature employed is given in Appendix F and a partial one below for Tables A-1 and A-2. Tridiagonal matrix coefficients are easily determined by recalling that a would be the coefficient of the i-1 term, b the i term, and c the i + 1 term and d the remaining terms. (See Appendix D.)

Nomenclature for Tables A-1 and A-2

$A1 = \alpha 1 \Delta t/(\Delta x1)^2$ - material 1

$A2 = \alpha 2 \Delta t/(\Delta x2)^2$ - material 2

$B1 = \alpha 1 \Delta t/(\Delta y)^2$ - material 1

$B2 = \alpha 1 \Delta t/(\Delta y)^2$ - material 2

$C1 = H1/\rho 1 C_p 1 T_B$ = heat distribution function (f(y)) for material 1

$C2 = H2/\rho 2 C_p 2 T_B$ = heat distribution function (g(y)) for material 2

$$E = \frac{k2\Delta x2}{k1\Delta x1}$$

$$F = [k2\Delta x1/k1\Delta x2]$$

$$G = \frac{k2\Delta x2\alpha 1}{k1\Delta x1\alpha 2}$$

$$\Phi = \left[C1 + \left(\frac{k2\Delta x2\alpha 1}{k1\Delta x1\alpha 2}\right) C2\right] \Bigg/ \left[1 + \frac{k2\Delta x2\alpha 1}{k1\Delta x1\alpha 2}\right]$$

$$\xi = \alpha 1 \left[1 + \frac{k2\Delta x2}{k1\Delta x1}\right] \Bigg/ \left[1 + \frac{k2\Delta x2\alpha 1}{k1\Delta x1\alpha 2}\right]$$

$$\delta u_{yy} = u_{I1,j-1} - 2u_{I1,j} + u_{I1,j+1}$$

APPENDIX B

FINITE DIFFERENCE EQUATIONS APPLYING AS BOUNDARY CONDITIONS AT THE INTERFACE BETWEEN MATERIALS 1 AND 2

I. CONTINUOUS FLUX AND TEMPERATURE AT THE INTERFACE

Both temperature and heat flux must be continuous at an interface assumed to be in good thermal contact. Using the nomenclature adopted in this report, this is equivalent to saying that

(1) u^*_{I1} is continuous

and

(2) $$k1 \frac{(u^*_{I1,j} - u^*_{I1-1,j})}{\Delta x1} = k2 \frac{(u^*_{I1+1,j} - u^*_{I1,j})}{\Delta x2} \quad . \qquad (8)$$

Equation (8) can be used directly in the tridiagonal matrix since only the terms $u^*_{I1-1,j}$, $u^*_{I1,j}$, $u^*_{I1+1,j}$ are involved. Therefore, by rearranging Eq. (8), the coefficients a_{I1}, b_{I1}, c_{I1}, and d_{I1} can be specified as:

TABLE A-1

DIFFERENCE EQUATIONS FOR COMPOSITE (X-FIRST)

Difference Equation	Condition	Range	Comments
begin x-sweep			
1. $u^*_{1,j} = u^*_{0,j}$	left boundary material 1	$j = 1,\ldots,J-1$ $i = 1,2$	Symmetry (no flux)
2. $u^*_{i,j} - u_{i,j} = \frac{A1}{2}(u^*_{i+1,j} - 2u^*_{i,j} + u^*_{i,j-1})$ $+ \Delta t C1 + \frac{B1}{2}(u_{i,j+1} - 2u_{i,j} + u_{i,j-1})$	material 1	$j = 1,\ldots,J-1$ $i = 1,\ldots,I1-1$	PDE, implicit x
3a. $(u^*_{I1,j} - u^*_{I1-1,j})\frac{k1}{\Delta x1} = (u^*_{I1+1,j} - u^*_{I1,j})\frac{k2}{\Delta x2}$	Interface	$j = 1,\ldots,J-1$	a. Cont. flux
3b. $u^*_{I1,j} = u_{I1,j} + \Phi\Delta t + \xi\delta u_{yy}\frac{\Delta t}{2\Delta y^2}$ $+ \frac{A1\,(u^*_{I1-1,j} + (1+F)\,u^*_{I1,j} + (F)\,u^*_{I1+1,j})}{2(1+G)}$	Interface	$j = 1,\ldots,J-1$ $i = I1$	b. Cont. flux and PDE apply
4. $(u^*_{i,j} - u_{i,j}) = \frac{A2}{2}(u^*_{i+1,j} - 2u^*_{i,j} + u^*_{i-1,j})$ $+ \Delta t C2 + \frac{B2}{2}(u_{i,j+1} - 2u_{i,j} + u_{i,j-1})$	material 2	$j = 1,\ldots,J-1$ $i = I1+1,\ldots,I2-1$	PDE, implicit x
5. $u^*_{I2,j} = u^*_{I2-1,j}$	right boundary material 2	$j = 1,\ldots,J-1$ $i = I2, I2-1$	symmetry (no flux)
begin y-sweep (no heat source term)			
6. $km(u^{**}_{i,1} - u^{**}_{i,0}) = \Delta y q^* m/T_B$	material 1 or 2 $m = 1,2$	$i = 1,\ldots,I1-1,$ $I1+1,\ldots I2-1$ $j = 0,1$	inside boundary (incident fixed heat flux) ($q^* = q_r$ rest time) ($q^* = q_p$ pulse time)
7a. $u^{**}_{i,j} - u^*_{i,j} = \frac{Am}{2}(u^*_{i+1,j} - 2u^*_{i,j}$ $+ u^*_{i-1,j}) + \frac{Bm}{2}(u^{**}_{i,j+1} - 2u^{**}_{i,j}$ $+ u^{**}_{i,j-1})$	$m = 1,2$	$i = 1,\ldots,I1-1,$ $I1+1,\ldots,I2-1$ $j = 1,\ldots,J$	materials 1 or 2 excluding interface and right & left boundaries.
7b. $u^{**}_{I1,j} = u^*_{I1,j} + \frac{A1}{(1+G)}(u^*_{I1-1,j}$ $- (1+F)\,u_{I1,j} + (F)\,u_{I1+1,j})$ $+ \frac{\xi\Delta t}{2\Delta y^2}(u^{**}_{I1,j+1} - 2u^{**}_{I1,j} + u^{**}_{I1,j-1})$	interface	$i = I1$ $j = 1,\ldots,J$	PDE implicit y applies at interface if Eq. (3b) is used
8. $-km(u^{**}_{i,J} - u^{**}_{i,J-1}) = \Delta y h(u^{**}_{i,J})$	$m = 1,2$	$i = 1,\ldots,I1-1,$ $I1+1,\ldots,I2-1$ $j = J-1,J$	outside boundary (temp. dependent flow with liq. metal heat transfer coeff.)

TABLE A-2

DIFFERENCE EQUATIONS FOR COMPOSITE (Y-FIRST)

Difference equation	Conditions	Range	Comments
begin y-sweep			
1. $km\,(u^*_{i,1} - u^*_{i,0}) = \Delta y\ q_m/T_B$	materials 1 or 2 m=1,2	i=1,...I1-1, I1+1, ...,I2-1 j=1,0	inside boundary (incident fixed heat flux) (q* =qr for rest time) (q* =qp for pulse period)
2a. $u^*_{i,j} = u_{i,j} = \frac{Am}{2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) + \Delta t\, Cm + \frac{Bm}{2}(u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1})$	m = 1,2	i=1,...,I1-1,I1+1, ...,I2-1 j=1,...,J	material 1 or 2 (excluding interface and left boundaries)
2b. $u^*_{I1,j} = u_{I1,j} + \frac{C1 + GC2}{(1 + G)}\Delta t + \frac{A1}{(1+G)}(u_{I1-1,j} - (1+F)u_{I1,j} + (F)u_{I1+1,j}) + \frac{\xi\Delta t}{2\Delta t^2}(u^*_{I1,j-1} - 2u^*_{I1,j} + u^*_{I1,j+1})$	interface	i = I1	applies at interface if Eq. (6b) is used
3. $km\,(u^*_{i,J} - u^*_{i,J-1}) = \Delta y h(u^*_{i,J})$	m = 1,2	i = 1,...I1-1, I1+1 ...,I2-1 j = J-1,J	outside boundary (temperature dependent flux with liquid metal heat transfer coeff.)
begin x-sweep (no heat source term)			
4. $u^{**}_{1,j} = u^{**}_{0,j}$	material 1 left boundary	j = 1,...,J-1 i = 0,1	symmetry (no flux)
5. $u^{**}_{i,j} - u^*_{i,j} = \frac{A1}{2}(u^{**}_{i+1,j} - 2u^{**}_{i,j} + u^{**}_{i-1,j}) + \frac{B1}{2}(u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1})$	material 1	j = 1,...,J-1 i = 1,...,I1-1	PDE, implicit x
6a. $(u^{**}_{I1,j} - u^{**}_{I1-1,j})\frac{k1}{\Delta x1} = (u^{**}_{I1+1,j} - u^{**}_{I1,j})\frac{k_2}{\Delta x2}$	interface	j = 1,...,J-1 i = I1	a. continuous flux
6b. $u^{**}_{I1,j} = u^*_{I1,j} + \frac{\xi\delta u^*_{yy}\Delta t}{2\Delta y^2} + \frac{A1}{2\,(1+G)}(u^{**}_{I1-1,j} + (1+F)\,u^{**}_{I1,j} + (F)\,u^{**}_{I1+1,j})$		j=1,...,J-1 i = I1	b. continuous flux and PDE
7. $u^{**}_{i,j} - u^*_{i,j} = \frac{A2}{2}(u^{**}_{i+1,j} - 2u^{**}_{i,j} + u^{**}_{i-1,j}) + \frac{B2}{2}(u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1})$	material 2	j = 1,...,J-1 i = I1+1,...I2-1	PDE, implicit x
8. $u^{**}_{I2,j} = u^{**}_{I2-1,j}$	material 2 right boundary	j = 1,...,J-1 i = I2-1,I2	symmetry (no flux)

$$a_{I1} = -1$$

$$b_{I1} = 1 + \frac{k2\Delta x1}{k1\Delta x2}$$

$$c_{I1} = -\frac{k2\Delta x1}{k1\Delta x2}$$

$$d_{I1} = 0 \quad . \tag{9}$$

The stability and convergence of the ADI procedure appeared to depend on the choice of $\Delta x1$ and $\Delta x2$ for a given k1 and k2. If values of $\Delta x2$ were selected such that

$$\frac{k1}{\Delta x1} \cong \frac{k2}{\Delta x2} \quad , \tag{10}$$

the ADI technique was convergent and stable. Consequently, an alternate form of the interface condition was developed to keep the PDE itself continuous at the interface.

II. CONTINUOUS FLUX AND TEMPERATURE WITH MODIFIED PDE AT THE INTERFACE

By utilizing the technique suggested by Carnahan, Luther, and Wilkes,[7] one can develop appropriate finite difference equations for the boundary between material 1 and 2 for our case. Following the conventions of the model, the dimensionless temperature at position I1-1 in material 1 can be approximated by a Taylor expansion as

$$u_{I1-1,j} \cong u_{I1,j} - \Delta x1 \left(\frac{\partial u}{\partial x}\right)_{I1^-} + \frac{(\Delta x1)^2}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_{I1^-} + \ldots \tag{11}$$

by solving Eq. (11) for $(\partial^2 u/\partial x^2)_{I1^-}$, one gets

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{I1^-} \cong \frac{2}{(\Delta x1)^2}\left[u_{I1-1,j} - u_{I1,j} + \Delta x1 \left(\frac{\partial u}{\partial x}\right)_{I1^-}\right] \quad . \tag{12}$$

Using the finite difference equation for $(\partial^2 u/\partial y^2)$ and $\partial u/\partial t$

$$(\partial^2 u/\partial y^2) \cong \frac{1}{\Delta y^2}\left[u_{I1,j+1} - 2u_{I1,j} + u_{I1,j-1}\right] \tag{13}$$

$$(\partial u/\partial t) \cong \frac{1}{\Delta t}\left[u^*_{I1,j} - u_{I1,j}\right]$$

$$u^* \text{ at new time } t + \Delta t \quad . \tag{14}$$

Likewise for material 2, Eqs. (11), (12), (13), and (14) can be rewritten as,

$$u_{I1+1,j} \cong u_{I1,j} + \Delta x2 \left(\frac{\partial u}{\partial x}\right)_{I1^+} + \frac{(\Delta x2)^2}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_{I1^+} \tag{15}$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{I1^+} \cong \frac{2}{(\Delta x2)^2}\left[u_{I1+1,j} - u_{I1,j} - \Delta x2 \left(\frac{\partial u}{\partial x}\right)_{I1^+}\right] \tag{16}$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right) \cong \frac{1}{\Delta y^2}\left[u_{I1,j+1} - 2u_{I1,j} + u_{I1,j-1}\right] \tag{17}$$

$$\left(\frac{\partial u}{\partial t}\right) = \frac{1}{\Delta t}\left(u^*_{I1,j} - u_{I1,j}\right) \quad . \tag{18}$$

By substituting into the differential equation,

$$\alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + C = \frac{\partial u}{\partial t} \quad ,$$

one can develop an expression for $\partial u/\partial t$ at the interface. For medium 1, using Eqs. (12), (13), and (14)

$$\alpha 1\left[\frac{2}{(\Delta x1)^2}\left(u_{I1-1,j} - u_{I1,j} + \Delta x1 \left(\frac{\partial u}{\partial x}\right)_{I1^-}\right) + \frac{1}{\Delta y^2}\left(u_{I1,j+1} - 2u_{I1,j} + u_{I1,j-1}\right)\right] + C1 = \left(u^*_{I1,j} - u_{I1,j}\right)\Big/\Delta t \quad . \tag{19}$$

Solving for $(\partial u/\partial x)_{I1^-}$, by defining

$$\delta u_{yy} \equiv u_{I1,j-1} - 2u_{I1,j} + u_{I1,j+1} \quad ,$$

Eq. (19) becomes

$$\Delta x1 \left(\frac{\partial u}{\partial x}\right)_{I1^-} = \frac{(\Delta x1)^2}{2\alpha 1 \Delta t}\left(u^*_{I1,j} - u_{I1,j}\right) - \frac{(\Delta x1)^2 C1}{2\alpha 1} - \frac{(\Delta x1)^2}{2(\Delta y)^2}\delta u_{yy} + u_{I1,j} - u_{I1-1,j} \quad . \qquad (20)$$

Similarly for medium 2, using Eqs. (16), (17), and (18)

$$-\Delta x2 \left(\frac{\partial u}{\partial x}\right)_{I1^+} = \frac{(\Delta x2)^2}{2\alpha 2 \Delta t}\left(u^*_{I1,j} - u_{I1,j}\right) - \frac{(\Delta x2)^2}{2\alpha 2} C2 - \frac{(\Delta x2)^2}{2(\Delta y)^2}\delta u_{yy} + u_{I1,j} - u_{I1+1,j} \quad . \qquad (21)$$

Applying the interface condition of continuous flux, viz,

$$k1 \left(\frac{\partial u}{\partial x}\right)_{I1^-} = k2 \left(\frac{\partial u}{\partial x}\right)_{I1^+} \quad . \qquad (22)$$

We can use Eqs. (20), (21), and (22) to eliminate $\left(\frac{\partial u}{\partial x}\right)_{I1^-}$ and $\left(\frac{\partial u}{\partial x}\right)_{I1^+}$ by just rearranging Eqs. (20 and (21).

$$k1 \left(\frac{\partial u}{\partial x}\right)_{I1^-} = \frac{k1\Delta x1}{2\alpha 1\Delta t}\left(u^*_{I1,j} - u_{I1,j}\right) - \frac{k1\Delta x1 C1}{2\alpha 1} - \frac{k1\Delta x1}{2(\Delta y)^2}\delta u_{yy} + \frac{k1}{\Delta x1}\left(u_{I1,j} - u_{I1-1,j}\right) \qquad (23)$$

$$k2 \left(\frac{\partial u}{\partial x}\right)_{I1^+} = -\frac{k2\Delta x2}{2\alpha 2\Delta t}\left(u^*_{I1,j} - u_{I1,j}\right) + \frac{k2\Delta x2 C2}{2\alpha 2} + \frac{k2\Delta x2}{2(\Delta y)^2}\delta u_{yy} - \frac{k2}{\Delta x2}\left(u_{I1,j} - u_{I1+1,j}\right) \quad . \qquad (24)$$

Equations (23) and (24) can be used to solve for $u^*_{I1,j}$.

$$\left[\frac{k1\Delta x1}{2\alpha 1\Delta t} + \frac{k2\Delta x2}{2\alpha 2\Delta t}\right] u^*_{I1,j} = \left[\frac{k1\Delta x1}{2\alpha 1\Delta t} + \frac{k2\Delta x2}{2\alpha 2\Delta t}\right] u_{I1,j} + \left[\frac{k1\Delta x1 C1}{2\alpha 1} + \frac{k2\Delta x2 C2}{2\alpha 2}\right] + \left[\frac{k1\Delta x1}{2(\Delta y)^2} + \frac{k2\Delta x2}{2(\Delta y)^2}\right]\delta u_{yy} - \frac{k1}{\Delta x1}\left[u_{I1,j} - u_{I1-1,j}\right] - \frac{k2}{\Delta x2}\left[u_{I1,j} - u_{I1+1,j}\right] \quad . \qquad (25)$$

By simplifying Eq. (25),

$$u^*_{I1,j} = u_{I1,j} + \Phi\Delta t + \frac{\xi\,\Delta t\,\delta u_{yy}}{\Delta y^2} + \left[\frac{u_{I1-1,j} - u_{I1,j}\left[1 + \frac{k2\Delta x1}{k1\Delta x2}\right] + u_{I1+1,j}\left[\frac{k2\Delta x1}{k1\Delta x2}\right]}{\frac{(\Delta x1)^2}{2\alpha 1\Delta t}\left[1 + \frac{k2\Delta x2}{k1\Delta x1}\frac{\alpha 1}{\alpha 2}\right]}\right] \qquad (26)$$

where

$$\Phi \equiv \left[C1 + \frac{k2\Delta x2\,\alpha1}{k1\Delta x1\,\alpha2} C2\right] \Big/ \left[1 + \frac{k2\Delta x2\,\alpha1}{k1\Delta x1\,\alpha2}\right] \quad (27)$$

$$\xi = \alpha1 \left[1 + \frac{k2\Delta x2}{k1\Delta x1}\right] \Big/ \left[1 + \frac{k2\Delta x2\,\alpha1}{k1\Delta x1\,\alpha2}\right] . \quad (28)$$

Equation (26) is similar to the explicit difference equation presented by Arpaci.[2]

For the case of no heat generation, C1 = C2 = 0; $\Delta x1 = \Delta x2 = \Delta x$; and only one direction dependence for u, i.e., $\delta u_{yy} = 0$, u^* becomes

$$u^*_{I1,j} = u_{I1,j} + \frac{2\alpha1\Delta t}{\Delta x^2} \frac{\left[u_{I1-1,j} - u_{I1,j}\left(1 + \frac{k2}{k1}\right) + u_{I1+1,j}\left(\frac{k2}{k1}\right)\right]}{\left[1 + \frac{k2\alpha1}{k1\alpha2}\right]} . \quad (29)$$

By multiplying the numerator and denominator of the second term on the right-hand side of Eq. (29) by k1/k2 and rearranging, one gets,

$$u^*_{I1,j} = u_{I1,j} + \frac{2\alpha1\Delta t}{\Delta x^2} \frac{\left[u_{I1+1,j} - u_{I1,j}\left(1 + \frac{k1}{k2}\right) + u_{I1-1,j}\left(\frac{k1}{k2}\right)\right]}{\left[\frac{k1}{k2} + \frac{\alpha1}{\alpha2}\right]} , \quad (30)$$

which corresponds to Eq. (7.67) presented by Carnahan et al.[7] on page 463. If both materials are the same, $\alpha1 = \alpha2 = \alpha$; k1 = k2 = k and,

$$u^*_{I1,j} = u_{I1,j} + \frac{\alpha\Delta t}{\Delta x^2}\left(u_{I1+1,j} - 2u_{I1,j} + u_{I1-1,j}\right) , \quad (31)$$

which is in standard explicit form for a homogeneous system.

Using implicit formulation in order to implement this algorithm in the current ADI code, one can show that

$$u^*_{I1,j} = u_{I1,j} + \Phi^*\Delta t + \frac{\delta u_{yy}\,(\Delta t/2)\,\xi^*}{\Delta y^2}$$

$$+ \frac{\alpha1\Delta t/2}{(\Delta x1)^2} \frac{\left[u^*_{I1-1,j} - u^*_{I1,j}\left(1 + \frac{k2\Delta x1}{k1\Delta x2}\right) + u^*_{I1+1,j}\left(\frac{k2\Delta x1}{k1\Delta x2}\right)\right]}{\left[1 + \frac{k2\Delta x2\,\alpha1}{k1\Delta x1\,\alpha2}\right]} \quad (32)$$

with $\Phi = \Phi^*$, $\xi^* = \xi$.

(Note that again the heat source Φ^* is put in with full Δt, and $\Delta t/2$ is used for other time intervals.)

To determine the coefficients for the tridiagonal matrix, viz., a_{I1}, b_{I1}, c_{I1}, d_{I1}, we define the following quantities.

$$E \equiv \frac{k2\Delta x2}{k1\Delta x1}; \quad F \equiv \frac{k2\Delta x1}{k1\Delta x2}; \quad G \equiv \frac{k2\Delta x2\,\alpha1}{k1\Delta x1\,\alpha2} . \quad (33)$$

Note that $\delta u_{yy} = u_{I1,j-1} - 2u_{I1,j} + u_{I1,j+1}$ is defined at the old time t rather than $t + \Delta t$.

The first three terms on the right-hand side of Eq. (32) are used to specify d_{I1}, while the fourth term specifies a_{I1}, b_{I1}, and c_{I1}, along with the left-hand side of Eq. (32). Consequently,

$$a_{I1} = \frac{-2\alpha1\Delta t/2}{(\Delta x1)^2(1 + G)} \quad (34)$$

$$b_{I1} = 1 + \frac{2\alpha1\Delta t/2\,(1 + F)}{(\Delta x1)^2\,(1 + G)} \quad (35)$$

$$c_{I1} = \frac{-2\alpha1\Delta t/2(F)}{(\Delta x1)^2\,(1 + G)} \quad (36)$$

$$d_{I1} = u_{I1,j} + \frac{\Delta t\ (C1 + GC2)}{(1 + G)} + \frac{\Delta t \alpha 1\ (1 + E)}{2(1 + G)\Delta y^2}\left[u_{I1,j-1} - 2u_{I1,j} + u_{I1,j-1}\right] \quad (37)$$

(in the Madcap code $\alpha 1$ = D1 and $\alpha 2$ = D2) .

In the ADI scheme, we also need an equation to allow us to implicitly calculate $u_{I1,j}$ at the interface when sweeping in the y-direction. Since Eq. (25) is an equivalent form of the PDE applying at i = I1 (interface), it can be rewritten implicit in y and explicit in x. Equation (26) thus can be restructured as

$$u^*_{I1,j} = u_{I1,j} + \Phi\Delta t + \frac{\xi\Delta t}{2\Delta y^2}\left[u^*_{I1,j-1} - 2u^*_{I1,j} + u^*_{I1,j+1}\right]$$

$$+ \frac{2\alpha 1 \Delta t/2}{(\Delta x1)^2}\,\frac{\left[u_{I1-1,j} - u_{I1,j}\left[1 + \frac{k2\Delta x1}{k1\Delta x2}\right] + u_{I1+1,j}\left[\frac{k2\Delta x1}{k1\Delta x2}\right]\right]}{\left[1 + \frac{k2\Delta x2\,\alpha 1}{k1\Delta x1\,\alpha 2}\right]}\ , \quad (38)$$

which is similar to Eq. (32). Again we can solve for the tridiagonal coefficients using Eq. (33) to define terms.

$$u^*_{I1,j} = u_{I1,j} + \frac{\Delta t\left[C1 + GC2\right]}{(1 + G)}$$

$$+ \frac{\Delta t\,\alpha 1}{(\Delta x1)^2\ (1 + G)}\left[u_{I1-1,j} - (1 + F)\ u_{I1,j} + (F)u_{I1+1,j}\right]$$

$$+ \frac{\xi\Delta t}{2\Delta y^2}\left[u^*_{I1,j-1} - 2u^*_{I1,j} + u^*_{I1,j+1}\right] \quad (39)$$

$$\xi = \frac{\alpha 1\,(1 + E)}{(1 + G)} \quad (40)$$

$$a_{I1} = -\frac{\xi\Delta t}{2\Delta y^2} = -\frac{\alpha 1\ (1 + E)\Delta t}{(1 + G)(2\Delta y^2)} \quad (41)$$

$$b_{I1} = 1 + \frac{\xi\Delta t}{\Delta y^2} = 1 + \frac{\alpha 1(1 + E)\Delta t}{(1 + G)\Delta y^2} \quad (42)$$

$$c_{I1} = -\frac{\xi\Delta t}{2\Delta y^2} = -\frac{\alpha 1(1 + E)\ \Delta t}{(1 + G)(2\Delta y^2)} \quad (43)$$

$$d_{I1} = \frac{\Delta t\,\alpha 1}{(\Delta x1)^2\ (1 + G)}\left[u_{I1-1,j} - (1 + F)\ u_{I1,j} + (F)u_{I1+1,j}\right] + u_{I1,j} + \frac{\Delta t\ (C1 + GC2)}{(1 + G)} \quad (44)$$

APPENDIX C

ALTERNATING DIRECTION IMPLICIT METHOD (ADI)

The implementation of the ADI method as discussed in Appendix A has been considered by numerous authors (7,9,10,11), and consequently only a brief discussion is included here. The ADI technique when applied to a rectangular grid network avoids the step size limitations of an explicit method and also uses a tridiagonal coefficient matrix for rapid calculation of the temperature grid at any time step. The basic concept is to use two difference equations, each applied at half Δt steps.

Each difference equation is implicit in either the x or y direction. For example, solving the two-dimensional elliptic equation

$$\alpha \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] = \frac{\partial u}{\partial t} \quad (45)$$

would involve iterations using difference equations of the following form for an (i,j) grid. The x-sweep [implicit in x] is written as

$$\frac{u^*_{i,j} - u_{i,j}}{\Delta t/2} = \frac{(u^*_{i-1,j} - 2u^*_{i,j} + u^*_{i+1,j})}{\Delta x^2} + \frac{(u_{i,j-1} - 2u_{i,j} + u_{i,j+1})}{\Delta y^2} , \quad (46)$$

and the y-sweep [implicit in y] as

$$\frac{u^{**}_{i,j} - u^*_{i,j}}{\Delta t/2} = \frac{u^*_{i-1,j} - 2u^*_{i,j} + u^*_{i+1,j}}{\Delta x^2} + \frac{(u^{**}_{i,j-1} - 2u^{**}_{i,j} + u^{**}_{i,j+1})}{\Delta y^2} , \quad (47)$$

where

$u_{i,j}$ = value of $u_{i,j}$ at time t

$u^*_{i,j}$ = value of $u_{i,j}$ at $t + \Delta t/2$ (half time step)

$u^{**}_{i,j}$ = value of $u_{i,j}$ at $t + \Delta t$ (full time step).

Richtymer and Morton[3] have demonstrated that this form of the ADI method is unconditionally stable regardless of the choice of Δx, Δy, or Δt. Our particular problem has three additional complications:

(1) A heat source term C is present [Eq. (1)].

(2) An interface between two materials is present.

(3) The inside boundary condition is time dependent (pulsed flux).

All of the above can induce instabilities and/or inadequate convergence unless the difference equations applying at the interface and boundaries are properly formulated. (See Appendix B.) Consistency for the difference equations has been demonstrated if the heat source term is introduced at the full time step, i.e., $C\Delta t$ is introduced in either the x or y sweep and not at both half-time steps.[5] Systematic errors due to this procedure were eliminated by altering the sweeping sequence to xyyxxyyx ... .

APPENDIX D

FORMULATION OF THE TRIDIAGONAL ALGORITHM

The ADI technique inherently generates equations for each grid point involving 3 adjacent terms in the u matrix.

$$u_{i-1,j},\ u_{i,j},\ u_{i+1,j}$$

or

$$u_{i,j-1},\ u_{i,j},\ u_{i,j+1}\ . \qquad (48)$$

The coefficients a,b,c refer to i-1 (j-1), i(j), and i+1(j+1) terms, respectively, while d refers to the remaining terms. Furthermore the a,b,c coefficients would be for terms involving the new time step either u^{*} or u^{**} (see Table I). Thus, the tridiagonal matrix can be represented as

$$\begin{bmatrix} b_0Z_0 & c_0Z_1 & & & & \\ a_1Z_0 & b_1Z_1 & c_1Z_2 & & & \\ \cdots & \cdots & \cdots & & & \\ & a_iZ_{i-1} & b_iZ_i & c_iZ_{i+1} & & \\ & & & & & \\ & & & & a_nZ_{n-1} & b_nZ_n \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ \cdots \\ d_i \\ \vdots \\ d_n \end{bmatrix} . \qquad (49)$$

[Z] refers either to $u_{i,j}$, j fixed, or $u_{i,j}$, i fixed. The algorithm for solving the tridiagonal matrix is relatively straightforward. The matrix is swept from top to bottom and then from bottom to top to solve for [Z]. The following flow sheet depicts this procedure.[7]

TRIDIAGONAL PROCEDURE

START

↓

$$\beta_0 = b_0$$

$$\gamma_0 = \frac{d_0}{\beta_0}$$

↓

for i = 1 to N

$$\beta_i = b_i - \frac{(a_i\, c_{i-1})}{\beta_{i-1}}$$

$$\gamma_i = \frac{(d_i - a_i\, \gamma_{i-1})}{\beta_i}$$

↓

$$Z_N = \gamma_N$$

↓

for j = N-1 to 0

$$Z_j = \gamma_j - \frac{(c_j Z_{j+1})}{\beta_j}$$

↓

STOP

APPENDIX E

MADCAP V LISTING

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```
"CTR COMPOSITE HEAT FLOW MODEL"
"ALTERNATING DIRECTION IMPLICIT METHOD USED"
"Pulsed case"
"Isotropic and homogeneous properties assumed for each material"
"Modified Code with continous interface condition"
"Variable Specification"
     "T = temperature, °C"
     "TB = bulk lithium temperature, °C"
     "Cp = heat capacity, cal/g°C"
     "p = density, g/cm3"
     "h = heat transfer coefficient, cal/cm2sec°C"
     "k = thermal conductivity, cal/cm sec°C"
     "D = thermal diffusivity = K/PCp, cm2/sec"
     "Tp = burn time for pulse, Micro-sec"
     "Tr = test time, Micro-sec"
     "Δx1 = x-step size in Material 1"
     "Δx2 = x-step size in Material 2"
     "Δy = y-step size"
     "Δt = step size for time"
     "Time = actual time, sec"
     "Tprint = interval between prints Micro-sec"
     "Yw = Wall thickness, cm"
     "L1 = size of material 1 element, cm"
     "L2 = size of material 2 element, cm"
     "sub or postscripts 1 and 2 refer to two different Materials"
"    sub or postscript 3 refers to average value at interface"
```

01,01b "Differiential Equation (Rectangular coordinates)"

01,01c "$D(d^2u/dx^2+d^2u/dy^2)+C(y) = du/dt$"

01,01d "Dimensionless parameters"

01,01e "$u = (T-T_B)/T_B$"

01,01f "$A = D\Delta t/\Delta x^2$"

01,020 "$B = D\Delta t/\Delta y^2$"

01,021 "$C\Delta t = H\Delta t/\rho C\rho$"

01,022 "$Q\Delta y/R$ = incident heat flux"

01,023 "where:"

01,024 "Postscripts 1 and 2 refer to two different materials"

01,025 "postscripts r and p refer to rest and burn periods"

01,026 "For example,"

01,027 " H is the internal heat generation term, it can take on"

01,028 "values: Hr1(y),Hp1(y),Hr2(y),Hp2(y)"

01,029 "Likewise for Q: Qr1,Qp1,Qr2,Qp2"

01,02a "u* = dimensionless temperature at 1/2 time step"

01,02b "u** = dimensionless temperature at complete time step"

```
"sense 1 - on for trial data set"
"sense 2 - on for print out at each"
"sense 3 - on set generation terms to zero"
"sense 4 - on to set up plots"
"sense 5 - on to terminate the iteration"
"sense 6 - on to terminate iteration and initial plotting"
"sense 7 - on ask for new print interval"
"sense 8 - on to use old interface condition at I1"
"         -k1(du/dx) = -k2(du/dx) in finite difference form"
"    off to use modified interface condition at I1"
"         Continuous flux and PDE apply"
"If cont. flux and PDE are used at the interface then the"
"interface is included in the y sweep"
"sense 9 - on to use harmonic mean for interface,"
"    off for arithmetic area average"
u,u*,u** 0 to 110,0 to 110
Z,a,b,c,d 0 to 110
Gr1,Gr2,Gp1,Gp2,Hr1,Hr2,Hp1,Hp2 0 to 110
x,y 0 to 110
"Array assignment for plots"
"ΔT = T-T1, °C"
"ΔT1 = inside surface (plasma) temp. rise for material 1 at (i=1,j=0)"
"ΔT2 = inside surface (plasma) temp. rise for material 2 at (i=I2-1,j=0)"
"ΔT3 = inside surface (plasma) temp. rise at interface (i=I1,j=0)"
"ΔT4 = outside surface (lithium) temp. rise for material 1 (i=1,j=J)"
"ΔT5 = outside surface (lithium) temp. rise for material 2 (i=I2-1,j=J)"
"ΔT6 = outside surface (lithium) temp. rise at interface (i=I1,j=J)"
```

02,01c $\Delta T1, \Delta T2, \Delta T3, \Delta T4, \Delta T5, \Delta T6, t_{0 \text{ to } 500}$

02,01d $Cal_{0 \text{ to } 2000}$

02,01e $Yaxis_{0 \text{ to } 500}$

02,01f $D629_{0 \text{ to } 10}$

02,020 v = 0

02,021 $t_0, \Delta T1_0, \Delta T2_0, \Delta T3_0, \Delta T4_0, \Delta T5_0, \Delta T6_0 = 0$

02,022 (200 characters) Comp1,Comp2

02,023 for i = 0 to 110

02,024 $a_i, b_i, c_i, d_i, Z_i = 0$

02,025 $\Delta T1_i, \Delta T2_i, \Delta T3_i, \Delta T4_i, \Delta T5_i, \Delta T6_i = 0$

02,026   for j = 0 to 110

02,027 $u_{i,j}, u^*_{i,j}, u^{**}_{i,j} = 0$

02,028 fo is " Temperature Profiles ($T-T_B$, ^oC) "

```
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                 |    |    |    |    |    |    |    |    |    |    |
03.001           f1 is "i =     0         1         I1/2      I1        (I1+I2)/2     I2-1
03.002           f2 is " j"
03.003           Tstop = 1000000
03.004           if sense 1 is on         "Trial data set"
03.005                read console by " Δt = x    Δx1 = x   Δx2 = x   Δy = x ":Δt,Δx1,Δx2,Δy
03.006                read console by "L1 = x L2 = x Ywall = x": L1,L2,Y_w
03.007                read console by " k1= x   k2 = x ":k1,k2
03.008                read console by "Cp1= x   Cp2= x ":Cp1,Cp2
03.009                read console by " p1= x  p2 = x ":p1,p2
03.00a                read console by " D1= x  D2 = x ":D1,D2
03.00b                read console by " h = x":h
03.00c                read console by "Qr1 = x Qr2 = x "
03.00d    cont.            "Qp1 = x Qp2 = x": Qr1,Qr2,Qp1,Qp2
03.00e                read console by " COMP1 = x    COMP2 = x ":Comp1,Comp2
03.00f                read console by "Tprint(micro-sec) = x": Tprint
03.010                if sense 4 is on
03.011                     read console by "Tstop=x": Tstop
03.012           otherwise            "input data"
03.013                Δt=1000; Δx1=.05; Δx2=.005; Δy=.02
03.014                L1 = 1 ; L2 = .1 ; Y_w = 1
03.015                k1=.158; k2=.034
03.016                Cp1=.0731; Cp2=.178
03.017                p1=8.57; p2=3.96
03.018                D1=.25; D2=.0434
03.019                h = .14
03.01a                Qr1 = 23.82 ; Qr2 = 23.82
03.01b                Qp1=238.2; Qp2=238.2
03.01c                Comp1 = "Nb"
```

03,01d   Comp2 = "Al_2O_3 small h = .14 cal/cm^{2}sec$^{\circ}$C "

03,01e   Tprint = 10000

03,01f   Tstop = 3000000

03,020 T_s = 600   "$^{\circ}$C"

03,021 T_p = 10000   "Micro-sec"

03,022 T_r = 90000   "Micro-sec"

03,023 $Ip = [(T_p/\Delta t)]$

03,024 $It = [((T_p+T_r)/\Delta t)]$

03,025 $I1 = [(L1/(2\Delta x1) + .5)]$

03,026 $I2 = I1 + [(L2/(2\Delta x2) + .5)]$

03,027 $J = [(Y_w/\Delta y + .5)]$

03,028 $\Delta t = .000001\ \Delta t$ "conversion to sec from micro sec"

03,029 $A1 = D1(\Delta t/\Delta x1^2)$

03,02a $A2 = D2(\Delta t/\Delta x2^2)$

03,02b $B1 = D1(\Delta t/\Delta y^2)$

03,02c $B2 = D2(\Delta t/\Delta y^2)$

03,02d   if sense 9 is on   "harmonic mean"

03,02e $k3 = (2k1 \times k2)/(k1+k2)$

```
          |    |    |    |    |    |    |    |    |    |    |
          otherwise          "arithmetic area average"
               k3 = (k1×Δx1+k2×Δx2)/(Δx1+Δx2)
          Index = 1
          Delta = [(.000001 × Tprint/Δt + .5)]
          "internal heat generation functions as arrays"
          if sense 3 is off
               read card by "(d10)5": Points,Fract1,Fract2,Fract3,Fract4
               new card
               for i = 0 to Points
                    read card by "(d10)5": y_i,Qr1_i,Qp1_i,Qr2_i,Qp2_i
                    new card
               Degree = 2
               i0 = 1
               for j = 0 to J
                    ȳ = j(Δy)
                    for i = i0 to Points
                         if y_i ≥ ȳ
                              execute lagran(j,i,Degree,ȳ)
                              i0 = i
                              exit from loop
                         otherwise: loop back
          otherwise
               Fract1 = 1
               Fract2 = 1
               Fract3 = 1
               Fract4 = 1
               for j = 0 to J
```

```
                    Hr1j,Hp1j,Hr2j,Hp2j = 0
"Conversion from percent absorption to heat,cal/cm3sec "
for j = 0 to J
     Hr1j = Qr1×Hr1j/Δy
     Hr2j = Qr2×Hr2j/Δy
     Hp1j = Qp1×Hp1j/Δy
     Hp2j = Qp2×Hp2j/Δy
Qr1 = Fract1×Qr1
Qr2 = Fract2×Qr2
Qp1 = Fract3×Qp1
Qp2 = Fract4×Qp2
for i = 0 to I2          "Initial condition ui,j = 0"
     for j = 0 to J
          ui,j,u*i,j,u**i,j = 0
Time = 0
"begin. Of iterations for each time period Δt as n=1 to infinity"
"Code will proceed with one of two algorithims"
"    1 - if x and y profiles are important, 2-D ADI"
"    is used with entire heat source added at one"
"    half time step, and iteration sequence altered"
"    as XYYXXYYX in sweeping x and y arrays."
"    2 - if composite has very small x dimensions,"
"    i.e. if L1 and L2 are small compared to the"
"    thermal diffusion depths, only the y direction"
"    is used in the code, and a unidirectional ADI"
"    is run with average property values used"
"Test for parabolic (2D) or unidirectional dependence"
```

```
                 |    |    |    |    |    |    |    |    |    |    |
05,001          Twx1 = (L1/2)^2/D1

05,002          Twx2 = (L2/2)^2/D2

05,003          Twy1 = Yw^2/D1

05,004          Twy2 = Yw^2/D2

05,005          if sense 8 is off or (k1=k2) and (D1=D2) and (Δx1=Δx2)
05,006               Iomit = I2+1        "includes interface in computation"
05,007          otherwise
05,008               Iomit = I1          "excludes interface"
05,009          for n* = 2 to infinity
05,00a               if Model() = 1           "Parabolic ADI (2D) x and y Directions"
05,00b                    if Index ≤ Ip       "Counter to determine if in pulse or rest mode"

05,00c                         q1 = Qp1
05,00d                         q2 = Qp2
05,00e                         q3 = (q1×Δx1+q2×Δx2)/(Δx1+Δx2)
05,00f                    otherwise
05,010                         q1 = Qr1
05,011                         q2 = Qr2
05,012                         q3 = (q1×Δx2+q2×Δx2)/(Δx1+Δx2)
05,013                    if n* is even            "sweep x  first"
05,014                         execute eqone(0)
05,015                         for j = 1 to J-1
05,016                              if Index ≤ Ip

05,017                                   C1 = Hp1_j/(p1×Cp1×T_B)    "pulse period"

05,018                                   C2 = Hp2_j/(p2×Cp2×T_B)

05,019                              otherwise
05,01a                                   C1 = Hr1_j/(p1×Cp1×T_B)        "rest period"
```

```
                C2 = Kr2_j/(ρ2*Cp2*T_B)
            for i = 1 to I1-1
                execute eqtwo(i,j,A1,B1,C1,Δt,1)
            execute eqthree(I1,j,k1,k2,D1,D2,C1,C2,Δx1,Δx2,Δy,Δt,0)
            for i = I1+1 to I2-1
                execute eqtwo(i,j,A2,B2,C2,Δt,1)
            execute eqfive(I2)
            execute std(I2;a,b,c,d,z)
            for i = 0 to I2
                u*_i,j = z_i
        for i = 1 to I2-1 ∧ i ≠ Iomit            "begin y sweep"
            if i < I1: A=A1;B=B1;k=k1;q=q1      "material 1"
            if i = I1: q=q3; k=k3          "interface"
            if i > I1: A=A2;B=B2;k=k2;q=q2          "material 2"
            execute eqsix(0,Δy,q,k,T_B)
            for j = 1 to J-1
                if i ≠ I1
                    execute eqfour(i,j,B,A,0,0,0)
                otherwise
                    execute eqseven(I1,j,k1,k2,D1,D2,0,0,Δx1,Δx2,Δy,Δt,0)
            execute eqeight(J,Δy,h,k)
```

```
                    execute std(J;a,b,c,d,Z)
                    for j = 0 to J
                         u**_{i,j} = Z_j
          otherwise:              "Sweep y first"
               for i = 1 to I2-1 | i ≠ Iomit
                    if i < I1: A=A1;B=B1;K=k1;q=q1      "material 1"
                    if i = I1: q=q3;K=k3                "interface"
                    if i > I1: A=A2;B=B2;k=k2;q=q2           "material 2"
                    execute eqsix(0,Δy,q,K,T_B)
                    for j = 1 to J-1
                         if Index ≤ Ip
                              C1 = Hp1_j/(p1×Cp1×T_B)
                              C2 = Hp2_j/(p2×Cp2×T_B)
                         otherwise
                              C1 = Hr1_j/(p1×Cp1×T_B)
                              C2 = Hr2_j/(p2×Cp2×T_B)
                         if i < I1: C = C1
                         otherwise: C = C2
                         if i ≠ I1
                              execute eqtwo(i,j,B,A,C,Δt,0)
                         otherwise
                              execute eqseven(I1,j,K1,K2,D1,D2,C1,C2,ΔX1,ΔX2,ΔY,Δt,1)
                    execute eqeight(J,Δy,h,k)
                    execute std(J;a,b,c,d,Z)
                    for j = 0 to J
                         u*_{i,j} = Z_j
               for j = 1 to J-1          "begin of x sweep"
```

```
06,01c                                 execute eqone(0)
06,01d                                 for i = 1 to I1-1
06,01e                                      execute eqfour(i,j,A1,B1,0,0,1)
06,01f                                 execute eqthree(I1,j,k1,k2,D1,D2,0,0,Δx1,Δx2,Δy,Δt,1)
06,020                                 for i = I1+1 to I2-1
06,021                                      execute eqfour(i,j,A2,B2,0,0,1)
06,022                                 execute eqfive(I2)
06,023                                 execute std(I2;a,b,c,d,Z)
06,024                                 for i = 0 to I2
06,025                                      u**_{i,j} = Z_i
06,026                      "Missing values for u at [i=0,I1,I2; j=0,J]are assigned via B.C.'s"
06,027                      "These are not used in the computation of u(x,y,t)"
06,028                      u**_{0,0} = u**_{1,0}
06,029                      u**_{0,J} = u**_{1,J}
06,02a                      u**_{I2,0} = u**_{I2-1,0}
06,02b                      u**_{I2,J} = u**_{I2-1,J}
06,02c                      if k2 ≠ k1           "interface values at j=0,J"
06,02d                           Phi = (k2×Δx1)/(k1×Δx2)
06,02e                           u**_{I1,0} = ((Phi)u**_{I1+1,0} + u**_{I1-1,0})/(1 + Phi)
```

```
                    u**I1,J = ((Phi)u**I1+1,J + u**I1-1,J)/(1 + Phi)
                for i = 0 to I2
                    for j = 0 to J
                        ui,j = u**i,j
            otherwise           "Unidirectional (Yonly) dependence"
                A = (A1(L1)+A2(L2))/(L1+L2)        "average properties"
                B = (B1(L1)+B2(L2))/(L1+L2)
                k = (k1(L1)+k2(L2))/(L1+L2)
                if Index ≤ ID
                    q1 = Qp1
                    q2 = Qp2
                otherwise
                    q1 = Qr1
                    q2 = Qr2
                q = (q1(L1)+q2(L2))/(L1+L2)
                execute eqsix(O,Δy,q,k,TB)
                for j = 1 to J-1
                    if Index < ID
                        C1 = Hp1j/(p1×Cp1×TB)
                        C2 = Hp2j/(p2×Cp2×TB)
                    otherwise
                        C1 = Hr1j/(p1×Cp1×TB)
                        C2 = Hr2j/(p2×Cp2×TB)
                    C = (C1(L1)+C2(L2))/(L1+L2)
                    execute eqtwo(1,j,2B,O,C,Δt,O)
                execute eqeight(J,Δy,h,k)
                execute std(J,a,b,c,d,Z)
```

```
07,01c              for i = 0 to I2
07,01d                   for j = 0 to J
07,01e                        u(i,j) = Z(j)
07,01f        if Index = It: Index=1
07,020        otherwise: Index=Index+1                "end of Δt period"
07,021        if sense 2 is on; Interval = 0
07,022        otherwise: Interval = Ip
07,023        if (n-1) = Interval(mod Delta)
07,024                                                          "OUTPUT"
07,025             Time = (n-1)Δt
07,026             if sense 4 is on          "set up arrays for plotting"
07,027                  v = v+1
07,028                  t(v) = Time
07,029                  ΔT1(v) = u(0,0)*T(B)
07,02a                  ΔT2(v) = u(I2,0)*T(B)
07,02b                  ΔT3(v) = u(I1,0)*T(B)
07,02c                  ΔT4(v) = u(0,J)*T(B)
07,02d                  ΔT5(v) = u(I2,J)*T(B)
07,02e                  ΔT6(v) = u(I1,J)*T(B)
07,02f             if [(1000000 Time)] = [(Tprint)]
07,030                  new page
07,031                  print: date
07,032                  skip 4 lines
07,033                  print: "CTR COMPOSITE FIRST WALL"
07,034                  skip 2 lines
07,035                  if model() = 1
07,036                       Print: "Two dimensional ADI (x and y)"
07,037                  otherwise
07,038                       Print: "Unidirectional ADI (y-only)"
07,039                       Print: "average property values used"
```

```
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08.001                          skip 1 line
08.002                          print: "heat generation functions for pulse and rest mode"
08.003                          for j = 0 to J
08.004                              Print by "j=xx  y_j=x.x4  Hr1=x.x5+ee  "
08.005   cont.                          "Hr2=x.x5+ee  Hp1=x.x5+ee  "
08.006   cont.                          "Hp2=x.x5+ee": j,j×Δy,Hr1_j,Hr2_j,Hp1_j,Hp2_j
08.007                      new page
08.008                      print: date
08.009                      skip 4 lines
08.00a                      print: "CTR COMPOSITE FIRST WALL"
08.00b                      skip 2 lines
08.00c                      if model() = 1
08.00d                          print: "Two dimensional ADI (x and y)"
08.00e                      otherwise
08.00f                          print: "Unidirectional ADI (y only)"
08.010                          print: "average property values used"
08.011                      skip 1 line
08.012                      if sense 8 is on
08.013                          print: "cont. flux interface condition"
08.014                      otherwise
08.015                          print: "cont. flux and PDE at interface"
08.016                      if sense 9 is on
08.017                          print: "harmonic mean for k at interface"
08.018                      otherwise
08.019                          print: "arithmetic area average for k at interface"
08.01a                      print by " conductor(1) =  x       insulator(2) = x":Comp1,Comp2
08.01b                      skip 1 line
08.01c                      print: "incident flux - k(dT/dy)y = 0"
```

```
08.01d                print by "    pulse period qp1=x4.x4 qp2=x4.x4"
08.01e   cont.             "cal/sec cm2": Qp1,Qp2
08.01f                print by "    rest period qr1=x4.x4 qr2=x4.x4"
08.020   cont.             "cal/sec cm2": Qr1,Qr2
08.021                skip 1 line
08.022                print by "wall thickness (y direction) = x3.x4 cm":Yw
08.023                print by "element size material 1 (x direction) = x3.x4 cm":L1
08.024                print by "element size material 2 (x direction) = x3.x6 cm":L2
08.025                skip 1 line
08.026                print by "Δt = x5 micro-sec ":[(1000000Δt + .5)]
08.027                print by "Δx1 = x4.x5 cm ":Δx1
08.028                print by "Δx2 = x4.x5 cm ":Δx2
08.029                print by "Δy  = x4.x5 cm ":Δy
08.02a                skip 1 line
08.02b                print by "D1Δt/Δx1^2=x4.x5": A1
08.02c                print by "D2Δt/Δx2^2=x4.x5": A2
08.02d                print by "D1Δt/Δy^2 =x4.x5": B1
```

```
09,001                     print by "D2Δt/Δy² =x4.x5": B2
09,002                     skip 1 line
09,003                     print by "Pulse time = x.x5 sec  rest time = x.x5 sec":
09,004   cont.                  Tp/1000000,Tr/1000000
09,005                     skip 1 line
09,006                     print by "for material 1 k= x3.x5 cal/sec cm°C  "
09,007   cont.                  "Cp= x3.x5 cal/g°C  p= x3.x4 g/cm³  "
09,008   cont.                  "D1= x3.x5 cm²/sec": k1,Cp1,p1,D1
09,009                     skip 1 line
09,00a                     print by "for material 2 k= x3.x5 cal/sec cm°C  "
09,00b   cont.                  "Cp= x3.x5 cal/g °C  p= x3.x4 g/cm³  "
09,00c   cont.                  "D2= x3.x5 cm²/sec": k2,Cp2,p2,D2
09,00d                     skip 2 lines
09,00e                     print by "  h = x3,x4     effective heat transfer coeff,  cal/cm²sec°C ":h
09,00f                     skip 1 line
09,010                     print by "Grid size = (x(material 1) = x3 points, "
09,011   cont.                  "x(material 2) = x3 points) by (y = x3 points)":I1,(I2-I1),J
09,012                     new page
09,013                     print by "Time= x Micro-sec ": 1000000 Time
09,014                     skip 2 lines
09,015                     print : f0
09,016                     print: f1
09,017                     print: f2
09,018                     skip 2 lines
09,019                     I76 = [(I1/2)]
09,01a                     I77 = [((I1+I2)/2)]
```

```
        |    |    |    |    |    |    |    |    |    |    |
09,01b            for j = 0 to J
09,01c                 print by "xxx,,(,,sx.x4+ee)7,,": j,
09,01d  cont.               u_{0,j}*T_B, u_{1,j}*T_B, u_{I76,j}*T_B,
09,01e  cont.               u_{I1,j}*T_B, u_{I77,j}*T_B, u_{I2-1,j}*T_B,
09,01f  cont.               u_{I2,j}*T_B
09,020            if sense 7 is on
09,021                 read console by "Tprint=x": Tprint
09,022                 Delta = [(.000001*Tprint/Δt+.5)]
09,023            if sense 6 is on
09,024                 Tstop = 0
09,025       if 1000000*Time > Tstop and sense 4 is on
09,026            "plotting routine"
09,027            for m = 1 to 6
09,028                 for j = 1 to w
09,029                      if m = 1: Yaxis_j = ΔT1_j
09,02a                      if m = 2: Yaxis_j = ΔT2_j
09,02b                      if m = 3: Yaxis_j = ΔT3_j
09,02c                      if m = 4: Yaxis_j = ΔT4_j
09,02d                      if m = 5: Yaxis_j = ΔT5_j
09,02e                      if m = 6: Yaxis_j = ΔT6_j
09,02f                 D629_m = MAX_{n=1 to w}(Yaxis_n)
09,030            Tmax = MAX_{n = 1 to 6}(D629_n)
```

```
S629 = " TIME MICRO-SEC"
T629 = " T-Tm (C) "
U629 = "COMPOSITE-PULSED CASE"
execute cprime(Cal,2000,1,12)
execute csymbol(.1,9.5,.14,U629,0)
execute csymbol(.1,9.2,.14,Comp1,0)
execute csymbol(.1,9.0,.14,Comp2,0)
if Tmax <500 : Tmax = 500
otherwise: Tmax = 1000
Tstop = 1000000xTime
execute cscaler(0,Tstop,0,Tmax,0,10,0,10)
execute cplot(0,Tmax,3)
execute cplot(Tstop,Tmax,2)
execute cplot(Tstop,0,2)
execute caxis(0,0,0,10,S629,17)
execute caxis(0,0,90,-10,T629,-10)
for n = 1 to 6
     Symb = n
     for m = 0 to w
          if n = 1: Yaxis_m = AT1_m
          if n = 2: Yaxis_m = AT2_m
          if n = 3: Yaxis_m = AT3_m
          if n = 4: Yaxis_m = AT4_m
          if n = 5: Yaxis_m = AT5_m
          if n = 6: Yaxis_m = AT6_m
     q629 = 3
     for m = 0 to w
```

```
        |    |    |    |    |    |    |    |    |    |    |
0a,01c                                  execute cplot(1000000t_n,Yaxis_n,q629)
0a,01d                                  q629 = 2
0a,01e                             execute cnump((Tstop+.02),Yaxis_v,,14,Symb,0,0)
0a,01f                        execute cempty(1,4)
0a,020                        stop #1
0a,021                   if sense 5 is on; stop
0a,022              "Procedures"
0a,023              "eqone thru eqeight generate coefficients for the tridiagonal matrix"
0a,024              "    eqone - left hand boundary-material 1 (x-direction)"
0a,025              "    eqtwo - material 1 or 2, PDE at even At/2"
0a,026              "    eqthree - interface condition at I1 (x-direction)"
0a,027              "    eqfour - material 1 or 2, PDE at odd At/2"
0a,028              "    eqfive - right hand boundary-material 2 (x-direction)"
0a,029              "    eqsix - inside(plasma) side boundary (y-direction)"
0a,02a              "    eqseven - interface condition (PDE) for y sweep"
0a,02b              "    eqeight - outside, liquid metal heat transfer coeff, (y direction)"
0a,02c              "Std solves the tridiagonal matrix"
0a,02d              "lagran generates a lagrangian interpolation polynominal for"
0a,02e              "estimating discrete values of the heat generation term"
```

```
            "Model() determines with a 2-D or unidirectional solution"
            "will be used"
    (...    eqone(n;all)
            a_n=0;b_n=1;c_n=-1;d_n=0
    ...)
    (...    eqtwo(n,m,r,s,C,Δt,Test;all)
            (array)u
            if Test = 1
                 x = (u_{n,m+1}-2u_{n,m}+u_{n,m-1})
                 k* = n
            otherwise
                 x = (u_{n+1,m}-2u_{n,m}+u_{n-1,m})
                 k* = m
            a_{k*} = -r/2
            b_{k*} = 1+r
            c_{k*} = -r/2
            d_{k*} = C×Δt+(s/2)(x)+u_{n,m}
    ...)
    (...    eqthree(I1,j,k1,k2,D1,D2,C1,C2,Δx1,Δx2,Δy,Δt,Test2;all)
            (array)u,u*
            if sense 8 is on          "continuous flux at interface"
                 a_{I1} = -1
                 b_{I1} = 1+(k2×Δx1)/(k1×Δx2)
                 c_{I1} = -(k2×Δx1)/(k1×Δx2)
                 d_{I1} = 0
            otherwise           "continuous flux and PDE apply at interface"
```

```
                      |    |    |    |    |    |    |    |    |    |    |
0b,01b                     E = (k2×Δx2)/(k1×Δx1)

0b,01c                     F = (k2×Δx1)/(k1×Δx2)

0b,01d                     G = (k2×Δx2×D1)/(k1×Δx1×D2)

0b,01e                     a_I1 = -(D1×Δt)/(Δx1^2(1+G))

0b,01f                     b_I1 = 1 + (D1×Δt(1+F))/(Δx1^2(1+G))

0b,020                     c_I1 = -(D1×Δt×F)/(Δx1^2(1+G))

0b,021                     if Test2 = 0

0b,022                          H = D1(1+E)×(Δt/2)×(u_I1,j-1 -2u_I1,j +u_I1,j+1)/(Δy^2)

0b,023                          H2 = u_I1,j

0b,024                     otherwise

0b,025                          H = D1(1+E)×(Δt/2)×(u*_I1,j-1 -2u*_I1,j +u*_I1,j+1)/(Δy^2)

0b,026                          H2 = u*_I1,j

0b,027                     d_I1 = H2+(Δt(C1+G×C2)+H)/(1+G)

0b,028        ...)

0b,029        (...    eqfour(n,m,r,s,0,Δt,Test;all)

0b,02a                (array)u*

0b,02b                if Test = 1

0b,02c                     x = (u*_n,m+1 -2u*_n,m +u*_n,m-1)

0b,02d                     k* = n

0b,02e                otherwise

0b,02f                     x = (u*_n+1,m -2u*_n,m +u*_n-1,m)

0b,030                     k* = m

0b,031                a_k* = -r/2

0b,032                b_k* = 1+r
```

```
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                      |   |   |   |   |   |   |   |   |   |   |
0b,033                c    = -r/2
                       K*
0b,034                d    = G×Δt+s(x)/2+u*
                       K*                   n,m
0b,035       ...)
0b,036       (...     eqfive(I2;all)
0b,037                a   = -1;b  =1;c  =0;d  =0
                       I2       I2     I2     I2
0b,038       ...)
0b,039       (...     eqsix(n,Δy,q,k,T ;all)
                                      B
0b,03a                a  = 0; b  = 1; c  = -1; d  = (q×Δy)/(k×T )
                       n       n       n       n             B


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                      |   |   |   |   |   |   |   |   |   |   |
0c,001       ...)
0c,002       (...     eqseven(I1,j,k1,k2,D1,D2,C1,C2,Δx1,Δx2,Δy,Δt,Test3;all)
0c,003                (array)u,u*
0c,004                E = (k2×Δx2)/(k1×Δx1)
0c,005                F = (k2×Δx1)/(k1×Δx2)
0c,006                G = (k2×Δx2×D1)/(k1×Δx1×D2)
0c,007                N = D1(1+E)/(1+G)
0c,008                P = (Δt×D1)/(Δx1^2(1+G))
0c,009                if Test3 = 1
0c,00a                     M = P(u       -(1+F)u     +(F)u       )
                                 I1-1,j         I1,j        I1+1,j
0c,00b                     M2 = u
                                 I1,j
0c,00c                otherwise
0c,00d                     M = P(u*       -(1+F)u*     +(F)u*       )
                                  I1-1,j          I1,j         I1+1,j
0c,00e                     M2 = u*
                                  I1,j
0c,00f                a   = -(N×Δt)/(2Δy^2)
                       I1
0c,010                b   = 1+(N×Δt)/(Δy^2)
                       I1
0c,011                c   = -(N×Δt)/(2Δy^2)
                       I1
0c,012                d   = M2+M+Δt(C1+G×C2)/(1+G)
                       I1
0c,013       ...)
0c,014       (...     eqeight(n,Δy,h,k;all)
0c,015                a  = -1;b =1+Δy×h/k;c =0;d =0
                       n       n           n     n
0c,016       ...)
0c,017       (...     std(n;a,b,c,d,Z)          "Tridiagonal matrix algorithm"
0c,018                (array)a,b,c,d,Z
0c,019                B,G  to 110
                         0
```

```
0c,01a          for m = 0 to 110
0c,01b               B_m,G_m = 0
0c,01c          B_0 = b_0
0c,01d          G_0 = d_0/B_0
0c,01e          for m = 1 to n
0c,01f               B_m = b_m-a_m c_m-1/B_m-1
0c,020               G_m = (d_m-a_m G_m-1)/B_m
0c,021          Z_n = G_n
0c,022          for m = n-1,n-2,...,0
0c,023               Z_m = G_m-c_m Z_m+1/B_m
0c,024    ...)
0c,025    (...  lagran(j,i,Degree,ȳ;z11)
0c,026          (array)Hp1,Hp2,Hr1,Hr2,Gp1,Gp2,Gr1,Gr2,z,y
0c,027          for k = 1 to 4
0c,028               if k=1: z=Gp1
0c,029               if k=2: z=Gp2
0c,02a               if k=3: z=Gr1
0c,02b               if k=4: z=Gr2
0c,02c               c* = 1
0c,02d               for j* = i-1 to i+Degree-1
0c,02e                    if ȳ = y_j*
0c,02f                         Hp1_j = Gp1_j*
0c,030                         Hp2_j = Gp2_j*
0c,031                         Hr1_j = Gr1_j*
0c,032                         Hr2_j = Gr2_j*
0c,033                         exit from procedure
0c,034                    otherwise: c*=c*(ȳ-y_j*)
0c,035               z̄ = 0
0c,036               for i* = i-1 to i+Degree-1
0c,037                    t* = c*z_i*/(ȳ - y_i*)
0c,038                    for j* = i-1 to i+Degree-1
0c,039                         if i* = j*: loop back
0c,03a                         t* = t*/(y_i*-y_j*)
0c,03b                    z̄ = z̄+t*
0c,03c               if k=1: Hp1_j = z̄
0c,03d               if k=2: Hp2_j = z̄
```

```
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               |    |    |    |    |    |    |    |    |    |    |
0d,001                   if k=3: Hr1_j = z̄
0d,002                   if k=4: Hr2_j = z̄
0d,003     ...)
0d,004     (...     model(none; all)
0d,005              Tbarx = (Tvx1+Tvx2)/2
0d,006              Tbary = (Tvy1 + Tvy2)/2
0d,007              if Tbarx < .01Tbary
0d,008                   model() = 0          "unidirectional"
0d,009              otherwise
0d,00a                   model() = 1          "2-dimensional"
0d,00b     ...)
           $
```

APPENDIX F

NOMENCLATURE

Variable Specification

$A1 = \alpha 1 \Delta t/(\Delta x1)^2$

$A2 = \alpha 2 \Delta t/(\Delta x2)^2$

$B1 = \alpha 1 \Delta t/(\Delta y)^2$

$B2 = \alpha 2 \Delta t/(\Delta y)^2$

$C1 = H1/\rho 1 C_p 1 T_B$

$C2 = H2/\rho 1 C_p 1 T_B$

C_p = heat capacity, cal/g°C

$C(y) = H(y)/\rho C_p T_B$ designated as C1 or C2

D or α = thermal diffusivity = $k/\rho C_p$, cm^2/s

Δy = step size in both materials (y direction)

$\Delta x1$ = step size in material 1 (x direction)

$\Delta x2$ = step size in material 2 (x direction)

Δt = full time step

$\Delta T = T - T_B$ K or °C

$F = k2\Delta x1/k1\Delta x2$

h = heat transfer coefficient (liquid lithium, cal/cm^2 s °C)

H(y) = heat generation rate, cal/s cm^3, designated as Hr1, Hr2, Hp1, Hp2

I1 = number of grid pts in x-direction material 1

I2 = number of grid pts in x-direction material 2

J = number of grid pts in x-direction material 2

k = thermal conductivity, cal/s cm°C

L1 = size of element in material 1

L2 = size of element in material 2

ρ = density, g/cm^3

q or q_i = incident flux on the inside surface

T = temperature, K or °C

T_B = bulk Lithium temp., K or °C

τ_p = burn time for pulse, μ s or m s

τ_r = rest time, μ s of m s

u = dimensionless temperature = $(T-T_B)/T_B$

$u^*_{i,j}$ = dimensionless temp. 1/2 time interval

$u^{**}_{i,j}$ = dimensionless temp. full-time interval

Subscripts or postscripts

1-material 1

2-material 2

r-rest period

p-pulse (burn) period

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